**THESIS INFORMATION**

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| **Thesis title:** | **The solvability and properties of solutions of some nonlinear functional integrodifferential equations** |
| **Speciality:** | Mathematical Analysis |
| **Code:** | 62460102 |
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| **Academic year:** | 2016 - 2019 |
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# ABSTRACT

The thesis is devoted to the study of solvability and some properties of solutions of three forms for nonlinear functional integrodifferential equations. It consists of the 1-order, the *m*-order and the

(*m* + *n*)-order nonlinear functional integrodifferential equations in N variables with real values or with values in a general Banach space. In each problem studied, the existence of solutions, the uniqueness or the compactness of the set of solutions are proved. The main tools are the fixed point theorems together with the definitions of suitable Banach spaces and appropriate conditions for subsets to be relatively compact in these spaces. In order to illustrate the results obtained here, many examples are given.

# THE NEW RESULTS OF THESIS

Applying the fixed point theorems such as Banach’s theorem (also called contraction mapping principle), Schauder’s theorem, Krasnosel’skii’s theorem in the Banach spaces, using appropriate tools of nonlinear Analysis and giving the suitable assumptions, the new results are obtained. The thesis is composed of four chapters, each of them contains one aspect of the new results as follows.

(i) Constructing Banach spaces *X*1, *Xm*, *Xm*,*n* and establishing necessary and sufficient conditions for relatively compactness of subsets in *X*1, *Xm*, *Xm*,*n*. These techniques are presented in Chapter 1, they are useful to prove the existence of solutions of the equations investigated in Chapters 2, 3, 4.

(ii) Proving results on the existence, uniqueness and compactness of the set of solutions for the following 1-order nonlinear functional integrodifferential equation



where the functions are given, *E* is the general Banach space,  and the partial derivative of a function *u*(*x*) defined on Ω with respect to *x*1 is denoted by . These results are presented in Chapter 2. More precisely, Eq. (1) is considered in the cases of and , respectively. Under suitable conditions on the given functions involved therein, applying contraction mapping principle, the existence and uniqueness of the solution of (1) in *X*1 (with ) are proved. On the other hand, using Schauder’s theorem (with ) and Krasnosel’skii’s theorem (with ), the existence and the compactness of the set of solutions of Eq. (1) in *X*1 are obtained.

 (iii) Proving the existence and the compactness of the set of solutions of the following *m*-order 

nonlinear functional integrodifferential equation

 where , are given functions, *E* is the general Banach space. Denote by , the partial derivatives of order *i* (*i* = 1,…,*m*) of a function *u*(*x*) defined on Ω, with respect to the first variable. Under suitable assumptions on the functions *g*, *H*, *K*, as in Chapter 2 with , Krasnosel’skii’s theorem is applied in Chapter 3 to prove that the set of solutions of Eq. (2) is nonempty and compact in *Xm*.

(iv) Proving the existence and the compactness of the set of solutions of the following (*m* + *n*)-order nonlinear functional integrodifferential equation



with , where are given functions, *E* is also the general Banach space. Denote by ,the (*m* + *n*)-order partial derivative of a function *u*(*x*) defined on Ω, with respect to the first variable and the second variable  It is similar to Chapter 3, under suitable assumptions on *g*, *H*, *K*, Krasnosel’skii’s theorem is also applied in Chapter 4 to imply that the set of solutions of Eq. (3) is nonempty and compact in *Xm*,*n*.

(v) The existence and properties of solutions obtained as above also hold when *E* = , in spite of the fact that the assumptions of the main theorems are relaxed. In order to illustrate the results obtained here for (1), (2), (3), many examples are given in the cases *E* =  and *E* = *C*([0,1];).

# APPLICATIONS / APPLICABILITY IN PRACTICE OR UNRESOLVED PROBLEMS

Many practical problems in various fields of science, such as mechanics, physics, population dynamics and so on, can be formulated as integral equations or integrodifferential equations. These problems are extremely complex and remain unanswered partially or as a whole and hence it is necessary to conduct further research about them. The results obtained in the thesis can be a source of motivation for other PhD students to develop their research in the field of integral equations or integrodifferential equations.

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